Adapting Instruction for English Learners

English learners come to the classroom with all the variety of English speakers in regard to mathematics achievement. They may be at, behind, or ahead of grade expectations in mathematics. They may be gifted or eligible for special education services. They may have been born in the United States, or they may have arrived in this country very recently. They may speak one or more languages, and they may be literate in one or more languages other than English. They may be nearly fluent in English or have beginning or intermediate levels of understanding and production. They have in common one characteristic: They are all learning English.

Teachers can maximize success for English learners in the mathematics classroom by designing lessons that allow the students to access the content on several levels.

Getting To Know Your Students

Before school starts, check the cumulative folder on each student in your class to determine which ones are learning English. See if there is recent testing. Two types of testing are most useful: mathematics achievement and reading achievement levels. If no recent test information is available, you may instead administer a pre-course math test and ask English learners to write a dictated paragraph in English to assess their reading and writing skills. Make sure students understand that the testing is for diagnosis only, not for a numerical grade.

Use this data to help classify English learners as follows and design lessons and groupings that will benefit the various English learners in your classes.

Classification of English Learners

Low Mathematics Achievement/Low Reading Achievement

Who is this student?

- Student may be new to the class, school, or country.
- Student may have had inadequate schooling.
- Student may have moved a lot.
- Student may be unmotivated or have test anxiety.
- Low reading achievement may be depressing mathematics scores.
- Student may have gaps and holes in knowledge.
- Student may need special needs assistance.

What to do?

- Examine cumulative folder for other testing, notes, etc. Ask questions about language use and prior schooling.
- Delay any testing for a week or two. Help student feel comfortable in the class during that period of time.
- Administer mathematics achievement test and reading test, preferably in an individual setting.
- Assess this student at weekly intervals and monitor classroom work to determine if progress is being made.
- Look at the English Learner Lesson Notes for each chapter. Focus on utilizing visuals.
Low Mathematics Achievement/High Reading Achievement

Who is this student?
- Student may have been designated as an English learner because oral skills lag behind reading skills.
- Student may not test well in mathematics.
- Most students can make rapid progress in mathematics; a few may have learning difficulties that require the help of a specialist.

What to do?
- Assess mathematics achievement in a variety of ways.
- Concentrate on developing oral fluency by utilizing the group activities in the English Learner Lesson Notes.
- Focus on vocabulary specific to mathematics.
- Use a student’s reading ability to improve his or her math scores.

High Mathematics Achievement/Low Reading Achievement

Who is this student?
- Student has had good prior mathematics instruction.
- Mathematics is an area where this student can excel.
- Math achievement level may actually be higher than scores indicate. (Limited English reading skills affect mathematics achievement as well.)
- Word problems will be especially difficult.

What to do?
- Mathematics instruction should proceed at normal or near normal pace.
- Student should be involved in a systematic English language development program and intensive reading program outside of mathematics class.
- Spend part of each class period on mathematics vocabulary study.
- Provide student with a bilingual dictionary or math glossary and a mathematics text in the home language.
- Look at the English Learner Lesson Notes for those that are most useful.

High Mathematics Achievement/High Reading Achievement

Who is this student?
- Student may be ready for designation as a fluent English speaker.
- May need extra study in the specialized vocabulary of mathematics. Complex syntax may present obstacles to performance.
- Given systematic instruction, this student should be able to achieve at or above grade level.
What to do?

- Scan all of the suggestions in the English Learner Lesson Notes and progress through the ones the student needs as quickly as possible.
- Monitor carefully to make sure this student continues to progress at a reasonable pace.

Suggestions for Mathematics Teachers of English Learners

1. Allocate additional time for mathematics. Many students will be translating from English to their primary language and back again. When you ask questions, allow extra time for students to respond. Reading mathematics textbooks and understanding what is asked for in a word problem will be slower.

2. Use student’s background knowledge. Some English learners will have developed substantial background in mathematics; others will have very little. Find out what students know and then build on more.

3. Reduce the amount and sophistication of the English language used. This may be done by reordering the lessons in each chapter to begin with key vocabulary, followed by problems with a minimum of written English, followed by at least one word problem each day. Choose word problems that don’t rely on assumed background knowledge. Keep your questions direct and simple. Speak more slowly, avoid idioms and slang, be precise and concise, and use short sentences and simple vocabulary. Using hand gestures and pictures as well as words aids communication.

4. Develop predictable routines. Introduce one concept per day. Keeping the focus simple will aid students in understanding the point of the lesson.

5. Use different methods for getting a point across. Presenting concepts verbally and visually, with concrete examples and in abstract mathematical symbols, and using pictures, graphs, diagrams, and charts will enhance a student’s chance of understanding at least one of the presentations. As you introduce a new word, rule, or property, write it down.

6. Provide opportunities for English learners to interact with their English-speaking peers. Students who are learning a language need to hear native speakers using the language, and they need opportunities to use their new mathematics vocabulary in their speech and in their writing.

7. Provide opportunities for English learners to discuss their understandings with each other, confirm the homework assignments, or ask questions of each other in whatever language they may have in common. (Well-schooled parents can also help by explaining concepts/procedures in their home language.)
8. Allow English learners to demonstrate what they know in a variety of ways. When students first learn a language, they generally understand the spoken language before they can produce it themselves. Also, students may be able to read in English but not speak it themselves. Allow students to point, nod, gesture, draw a picture, or work math problems without words as they learn English.

9. Extend mathematics instructional time through homework, an extra class period, summer school, or tutoring. Many of the suggestions for English learners in this series can be carried out in collaboration with language arts teachers or other specialists, and are well-suited for discussion in a language arts class, English as a second language class, or in a tutorial.


Specific Suggestions
For each chapter you will find English Learner Lesson Notes to help you modify curriculum and instruction so that the content is accessible to English learners. The suggestions and activities in these sections are designed to (1) make the content more accessible for English learners; (2) explain mathematical concepts in a variety of ways; and (3) provide support for word problems and study skills. Much of the vocabulary study in the book may be review for some of your students. In that case, work as quickly as possible through the activities. For those students who need more systematic study, progressing through the activities as indicated will ensure that students understand basic terms prior to statewide testing that generally occurs toward the end of each school year. We recommend that if you have English learners in your classroom, you skim all the English Learner Lesson Notes so that you may use them as you need them.
English Learner Strategies for Math

Investigations by the Center for Applied Linguistics and others have shown that lesson plans that include the recursive steps of preparation, presentation, practice, expansion, and evaluation aid English learners in their comprehension and ownership of lesson material. Holt McDougal’s math programs support this lesson-plan structure.

Plan and Prepare

1 Make a Mathematics Word Wall
   Designate one section of your bulletin board for math terms and definitions.
   • Add new words and review words each day.
   • Keep frequently used words up all year.
   • Include mathematical and non-mathematical definitions of words with multiple meanings, such as base, leg, translation, and so on.
   • Include visual diagrams when appropriate.

2 Set Up a Vocabulary Resource File
   Collect photos, articles, and Web printouts that illustrate glossary terms.
   • Enlist students’ help in the hunt.
   • Keep items in tabbed folders organized by chapter for use in review.

3 Display Customary and Metric Measurements
   Display a poster showing customary and metric measurements.
   • Spell the units of measurement and list their abbreviations.
   • Include conversion factors.
   • Relate simple fractions to their equivalent decimals.

Focus and Motivate

1 Preview Vocabulary
   A list of key vocabulary appears at the beginning of every lesson.
   • Pronounce key terms as part of vocabulary preview and practice.
   • Show students the highlighted vocabulary words in the lesson.
   • Encourage students to keep personal vocabulary binders.
   • Refer students to the Multi-Language Visual Glossary for definitions.

2 Include a Language Learning Goal
   Research shows that teaching the language used to express concepts helps retain concepts as well as develop language ability.
   • Point out the phrases commonly used to express a concept.
   • Mention equivalent phrases. For example, less than or equal to and not more than are equivalent.

3 Connect to Prior Learning
   • The “Before” statement at the beginning of each lesson reminds students of work they have already done related to the lesson.
   • The “Now” statement explains how the upcoming lesson will develop or extend previous work.
4 **Start with a High-Interest Motivator**
- The “Why” statement at the beginning of each lesson and the photograph near the lesson title suggest a real-life example or exercise in the lesson that motivates the content.
- Read the suggested related example or exercise, without solving.

**Teach**

1 **Use an Investigating Activity**
Some lessons include optional activities for use before the lesson. These activities expose students to ideas that will be developed in the lesson.
- Activities involve manipulatives or visualizations that introduce math content with less emphasis on vocabulary.
- Have English learners work with English-proficient students
- Encourage students to speak or write a sentence that answers the question posed for the activity.

2 **Use Familiar Examples and Contexts**
Use examples that exist in your classroom when possible.
- Ask students to brainstorm examples to illustrate a concept.
- Connect math content to previously learned content.
- Connect math content to other fields of study.

3 **Simplify Structure, Not Content**
Use short sentences. Turn clauses into sentences. Speak slowly.
- Alternate a question with an answer.
- Do not water down vocabulary.
- Use the Key Concept summaries in the text as helping statements.

4 **Model Thinking Skills and Mathematics**
While demonstrating problem solving, narrate your thought process.
- Explicitly describe steps.
- Explain your rationale for each.

5 **Vary Instructional Methods**
Consider alternative ways of presenting the same concept, as some methods will work better for some students, and other methods will work better for others.
- Direct instruction is one option for presenting information.
- Cooperative learning gives students the opportunity to practice and develop language, which in turn helps them retain math concepts.
- Use graphics and manipulatives when appropriate.

6 **Weave In Language Lessons**
Support for understanding academic English is featured in the Teacher’s Edition and the *Multi-Language Visual Glossary.*
- Help students understand sentences that contain abstract words.
- Reinforce the language learning goals that were previewed.
7 Allow Student Questioning and Wait Time
Encourage discussion in the classroom.
- Question students to prompt thinking.
- As you prompt discussion, allow wait time. English learners need time to formulate their comments or questions.

8 Frequently Check Comprehension
Use simple techniques to check student understanding. If most students are not understanding a concept, do not move forward. Try a different method for presenting the same concept.
- Ask questions with true/false or yes/no answers, and have each student indicate his or her answer with thumbs up (true/yes) or thumbs down (false/no). Scanning the classroom allows you to quickly see who is understanding.
- Ask students questions with short-phrase, one-word, or single-number answers. Have them write the answer to display. Erasable slates can be made by attaching a transparency to card stock and providing students with dry erase markers.

9 Use Technology and Audio-Visual Media
*Animated Algebra* reinforces key content in an interactive format. Concepts can be reviewed cumulatively or by section.

**Practice and Apply**

1 Offer a Menu of Content-Rich Exercises
Exercises in the teaching resources reinforce the student text.
- Use A, B, and C levels of practice from the *Resource Books*.
- Use the activities from the *Activity Generator*.
- Clarify directions and demonstrate steps for key exercise sets.

2 Rewrite Word Problems
Written English sometimes avoids redundancy, so word problems may lack repetition that is helpful to English learners.
- Rewrite, or have students rewrite, word problems prior to solving them, using simple, active sentences to make the problem clear. Complex sentences can often be rewritten as two sentences.
- Replace, or have students replace, pronouns with nouns in a problem to clarify what each pronoun represents.

3 Provide Opportunity for Interaction and Communication
Allow the opportunity to practice language in low-risk situations.
- Pair or group students and provide guided practice so they can discuss problems and solve them cooperatively.
- Have students explain math concepts orally and in writing to demonstrate understanding and provide practice with spoken and written language.
4 Allow Native-Language Dialogue and Writing
Allow native-language work in notebooks and discussions. Display students’ native-language translations of math vocabulary.

5 Give Practice in Explaining and Describing
Students can demonstrate and describe problem solving.
- Have students write out problems and their solutions for the whole class.
- Have students explain the process they used for solving a problem.

6 Have Groups Solve Puzzles and Play Games
Include puzzles, design problems, and games to teach mathematical modeling.

Assess and Reteach

1 Use Word Banks and Demonstrate Common Responses
Demonstrate expected responses to frequently used prompts.
- For a set of questions, model the first answer, thinking it through aloud.
- Scaffold student answers by listing words they could use in a response.

2 Review Notes with Partners
Pair an English learner with an advanced student to share notes and ideas. The advanced student can explain or demonstrate his or her ways of understanding.

3 Use Ongoing and Alternative Assessments
Decrease the risk aspect of testing. Weave assessments into the lesson.
- Support English learners by allowing written or oral answers.
- Assess using a rubric.

4 Conduct Student Conferences and Interviews
Studies have shown the importance of having students reflect on their learning at the time of assessment. Listen to students discuss their work.
- Encourage metacognitive reflection by questioning students on what worked well.
- Use the language of critical thinking. If students express opinions, affirm that they are evaluating. If they notice patterns, tell them they are analyzing.
- Have students keep a learning log of the learning strategies that worked for them.

Principles of Success for Teaching English Learners
Holt McDougal Math program components help you succeed in teaching English learners in many ways.

1 Increase Comprehensibility: Improve understanding through language scaffolding and through verbal and nonverbal support.

2 Promote Interaction: Promote communication with pair and group activities.

3 Improve Thinking Skills and Study Skills: Give explicit skills instruction.

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Lesson 1.1

Preview Key Vocabulary  Have students set up a word wall. To provide the English learner with a visual glossary, create a word wall that can be used throughout the year. Give groups of 2 to 4 students one or more 8½-by-11 inch sheets of paper with a vocabulary word printed on it. Select the words from the Review Key Vocabulary at the end of the chapter. This will assist them by previewing upcoming vocabulary. Be sure to include terms commonly used such as cubed, squared, etc. Organize the groups as follows. One student finds the definition, another student writes on the paper, and one or two students find(s) examples or applications.

On their paper they need to (a) define the term using their text, dictionary, or Internet, (b) provide an example using the term, and (c) present it to the class. These words should be posted in a specific area of the room for easy reference.

Visual Clue  Variables are represented by letters, which stand for numbers that vary. Using parentheses around the variable before and after substitution will give English learners a visual clue of the process of substitution. Further, the use of parentheses will help the students use the correct order of operations after substitution.

In conjunction with the parentheses, use a different colored pen to denote the variable and the number after substitution.

As in Example 1 on page 2, be sure to write the \( n \) in a different color and to put parentheses around the 3 in the first step.

\[
13 \ n \\
13 (\ ) \\
13 (3)
\]

To emphasize the variable as a placeholder, leave the parentheses blank and have students come to the board or overhead and write in the number that should go into the parentheses.

Lesson 1.2

Group Activity  Set up groups of 2 to 4 students with English learners and English speakers. Give each group a large poster paper and a problem written on an index card. These problems should be similar to Examples 1–3 on pages 8 and 9 in that they should contain different types of grouping symbols. Exercises 13–18 on page 55 from the Chapter Review would be good problems to choose from. Ask students to include a written explanation of each step on the poster and the major grouping symbol be written in red or highlighted. These may be posted around the room for easy reference.

Calculators  Be sure to review the proper use of calculators as many English learners may not be familiar with scientific calculators. Students will blindly trust their machines and fail to think about the order of operations. Often, there are two different input styles available for calculators. English learners may be more comfortable with one style over another. These students need to know which they have and how to use it. It is strongly recommended to calculate several problems by hand and then lead an activity of calculating the same problems with a calculator. A major error students do not understand is how a calculator views a problem such as Exercise 4. Some calculators will subtract 2 from 8 first and not square it.

Focus On Reasoning

Verbal Clues  In mathematics, an always, sometimes, or never statement usually refers to some set of objects. To find out what that set of objects is, students should look for the word “for” followed by a phrase such as “all real numbers” or “consecutive integers”. Often specific values are given as a domain, as in the Examples of 1.7, pages 43–45. Also students may have difficulty recognizing the statement if it is phrased as part of a question, as in Exercise 47 on page 6: “For which whole number value(s) of \( x \)”. Show how these instances can be reworded as “for \( x = \)” statement.
Lesson 1.3

**Experiential Clue** Copy Exercises 3–12 from page 18 and give a copy to each student. Instruct students to cross out key phrases such as *more than*, *a number*, *difference*, etc., and replace them with the appropriate math symbols. It will be extremely beneficial for English learners to be able to write on the paper with the problem.

**Visual Aids** It is very important to have manipulatives available for the students when they begin word problems. Try to select problems such as Example 2 on page 16, which lend themselves to visual demonstration.

Lesson 1.4

**Group Activity** To demonstrate the meanings of the inequality symbols using experiential clues, draw inequalities on 8½-by-11 sheets of paper. Have one student stand in front of the room holding the inequality symbol. Ask other students to place themselves on the appropriate side of the student with regards to height. If the student is taller they will be on the greater than side, if shorter, the less than side. Extend the idea by having two students form a compound inequality. Use students of equal height to discuss equality.

**Vocabulary** There are many words and phrases to express inequalities. English learners often have difficulty with phrases such as “at least” or “at most.” In particular, it is important to distinguish words and phrases for strict and non-strict inequalities. Have students make a list of the four inequality symbols and write as many words and phrases associated with each.

Lesson 1.5

**Graphic Organizers** Require students to use a guess and check table while learning how to write equations. English learners must get the information out of the text and into a diagram, table, chart, or other concept organizer. Model drawing pictures, charts, tables, etc. and picking out the key information and vocabulary. A well-developed word wall will be of great benefit at this juncture.

Lesson 1.6

**Diagram** Encourage students to arrange their information into mapping diagrams such as the one at the end of Example 1 on page 35. Further, include both input and domain as a header and, similarly, both output and range. English learners need to see the connection between these words many times.

**Concept Map** To connect the idea between ordered pairs, input/output tables, and written rules, have students create a concept map for several of the problems like Exercises 14–21 on page 39. Emphasize the need to work from any one of the three types of information.

<p>| Input/Output |</p>
<table>
<thead>
<tr>
<th>Domain/Range</th>
<th>Algebraic Rule</th>
<th>Ordered Pair</th>
</tr>
</thead>
</table>

Lesson 1.7

**Group Activity** Set up groups of 2 to 4 students with English learners and English speakers and provide a large sheet of poster paper or an overhead sheet. Provide task cards that have either an algebraic rule \( y = 3x + 1 \), an input/output table, or a set of ordered pairs. On the poster paper each group should copy their task card, create a graph and come up with the missing two representations of the data. The posters or overheads should be presented at the end of class.

**Focus On Functions**

**Vocabulary** “Relation” and “function” have many meanings in English and are commonly misused and confused in mathematical use. All students, not just English learners, will benefit from a discussion of these. To distinguish between the mathematical meanings, give some examples students can easily recall. Familial examples are good ones to use: a mother can have more than one child so mother-child is a relation but not a function; a child can have only one biological mother so child-mother is a relation and a function.
Lesson 2.1

Preview Key Vocabulary  Create a word wall as described in the lesson note for Lesson 1.1. This type of visual glossary is very beneficial to the English learner.

Visual Glossary  A classroom poster of the Venn diagram below Example 1 on page 66 should be posted in the classroom. The use of visuals will help English learners overcome the language barrier. The teacher should ask students to write numbers in the categories in which they belong.

Class Activity  Give students playing cards as they walk into class. Red are negative and black are positive. Have them place the card at the appropriate place on a number line. This activity allows for a quick assessment of integers with little interference due to language.

Focus on Functions

Vocabulary  The terms “empty set,” “union,” “intersection,” and “complement,” although they have various meanings in English, may not be problems in this context. But “cross product” and “universal set” could be troublesome. Add all of these to the word wall and discuss any concerns the students may voice.

Lesson 2.2

Visual Aid  A large number line from -15 to 15 including halves should be posted on the board or where all can see and refer to it easily. This visual can be easily referred to for a variety of topics and helps provide visual clues for the English learner.

Group Activity  Set up groups of 2 to 4 students with English learners and English speakers and provide them with poster paper. Give them a task card with one of the properties of addition listed in the Key Concept on page 77. On the poster they need to define the property in words, give the algebraic statement and a numerical statement. Groups will present their posters to the class and the posters should be hung for the duration of the unit for easy reference. This activity will help the English learner by creating an opportunity to present material verbally and create visual references for the classroom.

Diagram  Have students think of their number lines as a vertical elevator. Negative numbers are below the ground and positive numbers are above the ground. The vertical movement of adding positive and negative numbers seems natural to students.

Lesson 2.3

Clue to Meaning  English learners will benefit from the use a simple acronym that will provide them with a clue to the procedure of subtracting negative numbers. Have them rewrite the statement using addition. Use the acronym CAO (copy, add, opposite) to remind them of this process. Students should be instructed to copy the number that follows the minus sign, replace the minus with an addition sign, and change the number to its opposite. Having the acronym to refer to will provide the students with an experiential clue.

Lesson 2.4

Group Activity  Providing English learners with a graphic organizer such as the one in the Exploring Algebra Activity at the beginning of Lesson 2.4 on page 89 will help them organize the material. Have the students complete the table and write a rule that explains how to multiply integers. Emphasize the writing of the rule. To extend the topic ask them to write a rule for multiplying three integers and provide examples.

Students will have difficulty extending the pattern to a negative times a negative. You may need to lead individual group discussions.
**English Learner Lesson Notes continued**

**Pairs Activity** Pair English speakers and English learners and provide them with an overhead transparency and pen. Ask them to provide an example for each of the properties of multiplication. Show them the algebraic meaning and have them provide an example using numbers. Ask one pair to share and explain their example at the overhead.

**Lesson 2.5**

**Graphic Organizer** The use of rectangles will provide the English learner with a good visual clue for applying the distributive property as well as factoring. Use generic rectangles as shown in the introduction of Lesson 2.5 on page 96 and in the Exercises 40–42 on page 99: The two terms being multiplied are the length and width of the rectangle. Multiply to find the area of each section of the rectangle. The sum of the areas is equivalent to the product. This organizer will be beneficial when multiplying multi-term polynomials.

**Lesson 2.6**

**Jigsaw** Provide students with a graphic organizer like the one shown in the Concept Summary on page 105, except leave the explanations blank. Put students into groups of 2 to 4. Have each group write an overhead of a row or column. Groups should present their description of the rule while the rest of the class fills in their notes. Having English learners write the rules in their own words and present the material will greatly enhance their comprehension by increasing their interaction.

<table>
<thead>
<tr>
<th>Expression</th>
<th>$a + b$</th>
<th>$a + b$</th>
<th>$a \cdot b$</th>
<th>$\frac{a}{b}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Positive if ...</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Negative if ...</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Zero if ...</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Build on Past Knowledge** While explaining how to simplify Exercises 33–41 on page 106 it is important that you emphasize how these fractions can be written as two fractions. Show them how $\frac{6x}{2} - \frac{14}{2}$ equals $\frac{6x - 14}{2}$. Students know how to simplify one term fractions and will be able to make the connection to more complex fractions.

**Overhead Transparency** Create an overhead with a pattern showing how dividing is the same as multiplying by multiplicative inverse. Ask students to write an analogy for dividing. Use a different color for the divisor and the multiplicative inverse to provide a visual clue to the English learner.

Possible overhead:

<table>
<thead>
<tr>
<th>Dividing</th>
<th>Multiplying</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{8}{4} = 2$</td>
<td>$2 = (8)\left(\frac{1}{2}\right)$</td>
</tr>
<tr>
<td>$\frac{9}{3} = 3$</td>
<td>$3 = (9)\left(\frac{1}{3}\right)$</td>
</tr>
<tr>
<td>$\frac{10}{5} = 2$</td>
<td>$2 = (10)(?)$</td>
</tr>
</tbody>
</table>

**Lesson 2.7**

**Diagram** To provide English learners with a visual clue for square roots, have students factor the radicand. Ask them to circle the pairs of factors and remove the pair as one from the root. Those factors without pairs must stay inside the root.

**Focus on Operations**

**Illustrate** To illustrate the idea of approximating cube roots, draw a number line with the integers represented by the perfect cube roots such as the cube root of 8 and the cube root of 27. Ask students to mark where they think the cube root of 15 would fit. Next, have them guess what numbers the cube root of 15 is between. Visualizing the cube root between 2 and 3 should help students see an approximation. As the students become more proficient, you may make the approximations more difficult.
Lesson 3.1
Vocabulary  Use alternate vocabulary to assist English learners in understanding inverse operations. Try using words such as undo or opposite. Discuss the inverse operations by pairing addition with subtraction and multiplication with division. Lead in a discussion of inverse operations by first recalling the terms additive inverse and multiplicative inverse from Chapter 2. Then write simple expressions such as \( x + a \) and \( ax \) on the board and explain how inverses (opposites, reciprocals) are used to isolate \( x \). Doing this without equations will help students grasp the concept of inverse operations.

Visual Clue  To provide a visual clue for English learners, show equations with a line down the equal signs signifying the two sides of the equation. This line will provide a clue for identifying the inverse operation(s) to be performed.

\[
x + 2 = 5
\]

Visual Clue  Use a different color for the inverse operation. This will lend itself to either horizontal or vertical format shown in Example 2 on page 137. The difference in color highlights the idea of the inverse operation or the “undoing” of terms.

\[
x - 12 = 3
\]

\[
12 + (Write \ these \ in \ red.)
\]

\[
x = 15
\]

Lesson 3.2
Pairs Activity  Pair students and provide them with a two-step problem. Have one student complete one step then pass the paper to their partner. The second student will complete one step and return the problem to the first student. Continue this process until the problem is complete. This increased interaction helps the English learner develop understanding and can provide one-on-one tutelage if needed.

Graphic Organizer  Provide students with an organizer that is shaped like a T-chart. On the left column of the chart students will do the work as in the left-hand column of Example 2 on page 144. On the right column students will write the steps they performed as in the right-column of Example 2 on pages 144. This process will help English learners clarify understanding of the vocabulary and mechanics of solving equations.

<table>
<thead>
<tr>
<th>Work</th>
<th>Explanation</th>
</tr>
</thead>
</table>

Focus on Reasoning
Vocabulary  “Reasonableness” is a hard word even for students having no problems with the English language. Discuss the word and suggest alternates to “reasonable,” such as “acceptable” and “satisfactory.” Perhaps “appropriate” is the closest in meaning: the answer is not only correct but it makes sense in the context of the original problem. Add “reasonableness” to the word wall.

Lesson 3.3
Demonstration  Exercise 39 on page 156 lends itself well to a demonstration. Bring in three posters and ask students to figure how to hang them according to the problem. Posting the posters on a board and writing the equation will help English learners visualize the problem. Make the extension to the distance between the posters being a variable.

Lesson 3.4
Overhead Transparency  Instead of showing how a problem is worked down, consider showing a solution worked up. Create an overhead of Example 2 on page 158. Begin by showing them only the last three steps. Now expose the last four steps and ask what changed. Note the inverse operation on the overhead in a different color. Finally, expose all the steps and ask what changed. Note the distribution in a different color. Building the problem up will simplify the problem for the English learner.
Lesson 3.5
Visual Demonstration  Ratios lend themselves to easy demonstrations. English learners will greatly benefit from any visual you can provide them. Bring in any number of items such as colored candy or marbles and have students write several ratios showing the number of green to blue, or green to total number of candies or marbles.

Visual Clue  To demonstrate that there are two equivalent ways to write a proportion, use words instead of numbers as shown in the Avoid Errors note on page 167. Write the proportion three times showing how two are equivalent and one is not.

\[
\frac{\text{miles}}{\text{hour}} = \frac{\text{miles}}{\text{hour}} \quad \frac{\text{hours}}{\text{miles}} = \frac{\text{hours}}{\text{miles}} \quad \frac{\text{hours}}{\text{miles}} \neq \frac{\text{miles}}{\text{hours}}
\]

Then students can add the variable and numbers corresponding to the units.

Lesson 3.6
Diagram  A “Z” diagram may help English learners visualize how to cross multiply proportions.

\[
\frac{4}{a} \times \frac{24}{30} \quad 4(30) = 24(a)
\]

Pictures  Bring in maps of the native countries of your English learners and use them in place of the map in Example 4 on page 174. This will empower the student and help provide a better understanding of the meaning of the numbers in the ratio within a familiar context.

Visual Clue  While writing proportions, use the same color for the number and terms that will be multiplied when you use the cross product. As in Example 1 on page 172, write the 8 and 15 in one color and the x and 6 in another. This color clue will help English learners grasp the concept of cross product.

Lesson 3.7
Vocabulary  Emphasize to the English learner the relationship between the words used in percent problems and math symbols. This will help them work through the word problems. On a poster or word wall show that is represents =, of represents multiplication and what represents a variable. This visual glossary will help greatly.

Hands-on Activity  To help English learners translate percent problems to mathematical statements, provide them with a list of problems such as those in Examples 2 and 3 on page 179. Ask students to cross out the words and put the math symbol directly above the word. In doing so, they will create an accurate math sentence.

Focus on Operations
Vocabulary  While there are only several models for percent of change exercises, the terminology in word problems may be confusing. Find examples in local newspapers and bring them to class. Besides “save n%,” “n% off,” and “n% savings,” look for less obvious phrases such as “n% price reduction,” “$x lower,” and “$x cash rebate.” Explain differences between these and “buy n and get 1 free” sales. Have students look for other examples and bring them to class. Make a display on a bulletin board.

Lesson 3.8
Visual Clue  While solving literal equations, write like terms in the same colors. This will provide the English learner with a clue to what terms may be combined. Furthermore, using a specific color such as red, or simply circling the variable being solved for will provide clues as to which variable to isolate. For example write the y in Example 1 on page 186 in red to emphasize this is the variable to be isolated.
Lesson 4.1

Visual Glossary While introducing the vocabulary used for graphing on a coordinate plane, create a poster or handout of a coordinate plane. Label important ideas such as quadrants, axes, intercepts, range, domain, etc. Add terms and examples as necessary, and use it as an easy reference or review.

Visual Clue Help the English learner make the connection between the coordinates in a table and on a graph by color coordinating x and y terms. In Example 3 on page 211, write the x terms in the table and the x-axis in one color, and the y terms in the table and the y-axis in another color.

Overhead Transparency While introducing coordinates, find some data of rainfall or temperature over a period of time from the countries the English learners are from. Then go through the steps of plotting the data. Having a graph that is relevant to them will keep their interest and foster a connection between you and them.

Focus on Graphing

Graphic Organizer English learners will benefit from the use of an organizer to summarize the results in this feature. Below is the beginning of a sample organizer.

<table>
<thead>
<tr>
<th>Name Transformation</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>horizontal translation</td>
<td>(x, y) → (x + h, y)</td>
</tr>
</tbody>
</table>

Lesson 4.2

Graphic Organizer Provide English learners with an organizer that will help them see the connection between the graph, the table, and the equation. Having a tool to structure notes allows the English learner to see the concepts with simpler text. Below is a sample organizer.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Table</th>
<th>Graph</th>
</tr>
</thead>
<tbody>
<tr>
<td>y = 2x + 4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Focus on Functions

Vocabulary The term discrete may cause some initial confusion. Use synonyms such as separate, individual, and distinct to help explain the meaning. The word continuous will probably be better understood in its mathematical context but add both words to the word wall.

Lesson 4.3

Tables Help the English learner see the connection between intercepts on the graph and in the equation by using tables with graphs. Circle the x and y intercepts in the table and the graph to highlight the connection.

y = 2x + 1

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>-3</td>
</tr>
<tr>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
</tr>
</tbody>
</table>

Visual Glossary This would be a good time to add a graph with a table to the poster or handout described in the lesson note for Lesson 4.1. Be sure to add examples of x- and y-intercepts. Circle the intercepts in the table and label them with coordinates on the graph to help the English learner see the connection.
**English Learner Lesson Notes**

**Word Problems** Have English learners go through the examples and exercises in the text to make lists of common quantities in word problems with possible units of measure. For example, from Example 5 on page 232, students can start the following list.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Units of measure</th>
</tr>
</thead>
<tbody>
<tr>
<td>elevation</td>
<td>feet</td>
</tr>
<tr>
<td>time</td>
<td>minutes</td>
</tr>
</tbody>
</table>

As students go through more and more problems they may add to the list.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Units of measure</th>
</tr>
</thead>
<tbody>
<tr>
<td>elevation, distance</td>
<td>feet</td>
</tr>
<tr>
<td>time</td>
<td>minutes, hours</td>
</tr>
</tbody>
</table>

**Lesson 4.4**

**Experiential Clue** To help students grasp the concept of positive and negative slope, have students interpret and explain graphs such as the one in Example 6 on page 242. One method for doing this would be to give groups of 2 to 4 students a graph to describe as in Exercises 38 and 39 on page 245. Ask the group to write a description of the graph emphasizing vocabulary. This group process will help English learners clarify their understanding of positive and negative slope by participating in conversation about the subject matter.

**Lesson 4.5**

**Inquiry-Based Activity** Set up groups of 2 to 4 students with English learners and English speakers. Provide them with a graphic organizer like the one shown below to work on the Exploring Algebra Activity on page 247. The organizer simplifies the activity by allowing the English learner to focus on the content using tables and graphs. Ask students to graph the lines, highlighting the y-intercepts, and drawing slope triangles. From their graphs, they should fill in their organizers. Finally, ask students to draw conclusions like the ones listed at the bottom of the activity.

<table>
<thead>
<tr>
<th>Line</th>
<th>Table</th>
<th>Graph</th>
<th>y-intercept</th>
<th>Slope</th>
</tr>
</thead>
</table>

**Focus on Graphing**

**Diagram** English learners will be more apt to learn the concept of finding the solution by graphing if it is presented in simpler text and with diagrams. Use a simplified graph, labeling the function \( f \) and the \( x \)-intercept. Emphasize the relationship between the \( x \)-intercept of the graph of a function \( f \) and the solution to the equation \( f(x) = 0 \)—that, to find the \( x \)-intercept, they need to solve the equation \( f(x) = 0 \).

**Lesson 4.6**

**Vocabulary** The term variation may be confusing at first because it is similar to the term variable. Be sure to stress that variation refers to how the variable \( y \) varies in relation to the variable \( x \). Introduce direct variation equations by focusing on the \( y \)-intercept of linear equations. Draw the connection between direct variation and the \( y \)-intercept being zero.

**Lesson 4.7**

**Tables** To help the English learner make the bridge from linear equation notation to function notation, continue to use tables. Consistently label the output column with both \( f(x) \) and \( y \). Providing them with a context clue will help them to make the connection between \( y \) and \( f(x) \) notation. A possible table may look like the one below. Be sure to include as many context clues as possible.

<table>
<thead>
<tr>
<th>( x ) (input)</th>
<th>( y ), ( f(x) ), (output)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>-1</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

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Lesson 5.1
Visual Clue  When introducing how to write equations in slope-intercept form, color code slope and intercept to provide a visual clue for the English learner. For example, in problems like Example 3 on page 290 denote \( b \) and the intercept coordinate in one color. Another important clue would be the slope triangle. Again, draw the slope triangle and \( m \) in a matching color to provide as many visual clues as possible.

Simplify the Vocabulary  Provide students with copies of word problems so they may write on them. Show how to write verbal models, and then ask students to do it on their own. Example 5 on page 291 demonstrates writing a verbal model before writing an equation. This method simplifies the text for English learners. In creating a verbal model, students only need to find specific information from the text.

Lesson 5.2
Diagram  When asking English learners to write equations from points, begin by using a graph. This will provide them with a diagram and visual clues of the equation. Students should be encouraged to draw slope triangles and find intercepts on their graphs before they try to write the equation.

Pairs Activity  Provide English learners with an opportunity to articulate their understanding by doing the following pair activity. Provide students with flash cards that have information needed to write the equation of a line on one side and the method to do so on the other. Use the Concept Summary on page 300 as a guide in making the cards. Be sure to include the diagram as a visual clue for the English learner. Ask students to discuss which method is applicable in the given situations. Be sure students focus on identifying the method and not the equation writing at this point.

Lesson 5.3
Diagram  Continue to use graphs as visual clues for the English learner. As in Example 2 on page 309, provide a graph for reference and include a slope triangle. For more clues, draw the slope triangle and the slope ratio in the same color, as well as the point on the graph and the one in the equation.

Group Activity  To enhance the English learners understanding of writing equations, have them participate in group activities. Arrange students into groups of 2 to 4. Provide each group with different sets of information. Choose from a graph, an equation in point intercept form, an equation in slope/intercept form, or some point/slope data. Require the groups to use their information to write lines in both forms, and draw a graph. Have the groups make a report to display on a poster or on a overhead.

Focus on Functions
Simplify the Vocabulary  Point out that the word arithmetic can be a noun or an adjective. Tell them the different pronunciations for the noun and adjective forms. Have them recite the following statement. “We use arithmetic to analyze arithmetic sequences.” Students may ask why these sequences are called arithmetic sequences. Point out that the naming convention is mostly to distinguish them from other types of sequences. Tell them that the label arithmetic sequence is not very descriptive and that a more descriptive name for these sequences would be a linear sequence.

Lesson 5.4
Visual Glossary  The English learner may have trouble with the vocabulary describing the different forms of equations. To assist them, create small posters that show the different forms in which lines may be written. Post them in a place where students can refer to them easily.
**Pairs Activity** Have English learners engage in conversations about the concepts to help them clarify their understanding. Try this exercise to work on writing equations of lines in different forms using various information. Designate one student as A and another as B. Assign each student a problem to solve. Once they are both done with their problem, have student A explain their solution to student B. Then have student B explain their solution to student A. This activity may be done with the entire class. For this lesson, focus on problems similar to Examples 2, 3, and 4 on pages 317 and 318.

**Lesson 5.5**

**Overhead Transparency** In order to provide many visual clues for the English learner, try demonstrating the concept of parallel and perpendicular lines with slope triangles. Prepare an overhead with several parallel lines on the same set of axis. Draw a slope triangle for each line and write the equation by each graph. Demonstrate how the slope triangles produce equal $m$ values. Furthermore, prepare an overhead similar to the diagram at the top of page 326. Again include slope triangles on the graphs and write the equations by each graph. Demonstrate how the slope triangles produce inverse $m$ values.

**Hands-on Activity** To build a strong relationship with the English learners and to teach the concept of perpendicular and parallel lines, use a map of the native city of an English learner. Form groups of 2 to 4 students and ask them to identify parallel and perpendicular streets. To extend the topic, ask them to find streets that are neither parallel nor perpendicular.

**Lesson 5.6**

**Diagram** Use graphs to demonstrate the ideas in this lesson. This will help the English learner overcome any obstacles presented by the vocabulary. For each topic such as positive correlation and negative correlation, use graphs like the ones used at the top of page 331. Having the diagram as a visual clue will greatly help the English learner relate positive slope to positive correlation and so forth.

**Hands-on Activity** To increase interaction and provide visual experiential clues for the English learner, try the following activity. Create a large graph on poster paper. Label the vertical axis height and the horizontal axis shoe size. Give each student a sticker to place at the coordinate that fits them. Then ask the class to fit a line to the class data. You may extend this activity by choosing different data to graph, separating the boys and girls information, or working in groups. The experiential aspect and the group work will help the English learner understand the concept.

**Lesson 5.7**

**Visual Glossary** To help English learners manage the vocabulary, provide them with clues on the walls or in their notes. Students may get confused when they are asked to find the zero of a function. To help students, post a sign that reminds them $y$ and $f(x)$ can be used interchangeably depending on equation type.

**Vocabulary** The words interpolate and extrapolate can easily be confused. Associate *inter* with inside and *extra* with outside. Plot the data and show that to interpolate is to look inside the range of the data and to extrapolate is to look outside the range of the data.

**Group Activity** To assist the English learners assimilate the main concepts and ideas presented in this chapter, provide them with the opportunity to interact, discuss, and listen in a group situation. Form groups of 2 to 4 English learners and English speakers. Ask them to make a poster of a multifaceted problem like those in the problem solving section on pages 345 and 346. The groups should work together to create the poster and then present them to the class. This will provide multiple opportunities for the English learner to interact in small groups, helping them to process the concepts. The presentation will be another visual aid to help their understanding.
Lesson 6.1

Pairs Activity  Provide activities that promote interaction between English speaking and English learner students to help English learners process concepts. To introduce the concept of inequalities, pair students and have them write situations that would result in an inequality. Exercises 3–5 on page 367 would be good problems to model this activity.

Study Guide  Provide English learners with worksheets that have word problems similar to Example 5 on page 366. Give extra space on the worksheets so students can mark up the problem in order to translate key phrases into algebraic symbols. Review key phrases from Lesson 1.4 in regards to expressing inequalities verbally.

Lesson 6.2

Graphic Organizer  Provide English learners with organizers to communicate the mathematical process. This will help students reach a deeper understanding of the mathematics. A double-sided organizer is a good tool. Have students do work on one side of the organizer and then explain the process on the other side. This allows English learners to separate the mathematics from verbalizing the process. For example, one side may show the steps to solve \(-3x > 6\). Then the other side should explain the process as “dividing both sides by \(-3\) and reversing the inequality.” Give students several different inequalities requiring multiplication and division, including those with multiplication and division of negative numbers.

Game  Use games to provide the English learner with the opportunity to interact and communicate about math. Create index cards with problems similar to Exercises 3–26 on page 374. Give each group a set of three cards and have them work through them. When the group is done, check their work and give them another set of three cards. This activity can be enhanced by awarding points for the number of sets each group works through. It is important to not provide answers and let the groups help themselves find solutions.

Lesson 6.3

Group Activity  Use group activities to provide English learners with a chance to interact and develop meaning through communication. Have students work in groups of 4. Give each group member a sheet of paper with one problem similar to those in Example 4 on pages 378. Have each person do one step and write an explanation before passing the paper clockwise to the next person in their group.

Experiential Clue  Students may have a difficult time visualizing the inequality statements in word problems similar to Example 5 on page 379. To assist English learners in writing the inequality, have them first write the related equation. Then they can turn it into an inequality. In Example 5, have students cross out “at most” from the original question. Building on their knowledge of writing equations will allow them to make the connection to writing inequalities more easily.

Focus on Graphing

Study Guide  Give English learners a tool to synthesize this material with what they have previously learned. Tell them that they are learning a graphical method for solving inequalities that they have previously solved algebraically. First solve Example 1 algebraically. Then solve it graphically.

\[
\begin{align*}
3x + 2 &> 8 \\
3x &> 6 \\
x &> 2
\end{align*}
\]

Algebraically  Graphically
\[
\begin{align*}
3x + 2 &> 8 \\
3x - 6 &> 0 \\
Graph & y = 3x - 6. \\
For \ what \ values \ of \ x \ is \ the \ graph \ above \ the \ x-axis? \\
x &> 2
\end{align*}
\]

Lesson 6.4

Pairs Activity  Communicating ideas helps English learners clarify understanding and concepts. Promote this type of interaction by grouping students in pairs for this activity. When introducing compound inequalities, use an overhead that
has several problems such as those in Example 1 on page 388. Give each pair only the verbal phrase or the graph. Ask the students to write the missing inequality. Have the students compare their responses before providing the solutions.

**Graphic Organizer** Provide English learners with an organizer to help them structure their thought process and make concepts more accessible by removing and simplifying text. For solving compound inequalities similar to those in Examples 3–5 on pages 389 and 390, use the following organizer.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Type/Method</th>
<th>Solution</th>
<th>Graph</th>
</tr>
</thead>
<tbody>
<tr>
<td>$6 &lt; x + 2 &lt; 9$</td>
<td>“and”</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Lesson 6.5**

**Visual Clue** Providing visual clues for English learners gives them a non-verbal link to important concepts. For example, a strong visual clue for students to isolate the expression within the absolute value symbols in order to solve the absolute value equation would be to highlight the expression including the absolute value symbols. Emphasize isolating the highlighted material before rewriting as two equations. Have students use this visual clue to emphasize the procedure.

**Pairs Check** Pair students and provide them with problems similar to Exercises 15–20 on page 401. Have one student complete one step of a problem and then pass the paper to their partner. The second student will complete the next step and return the problem to the first student. Have them continue this process until the problem is solved.

**Focus on Functions**

**Visual Clues** While introducing the parent function for absolute value functions, color code the types of transformations to provide visual clues for the English learner. For example, use red to denote the $h$ in the equation and a horizontal shift, green to denote the $k$ in the equation and a vertical shift, blue to denote the $a > 0$ in the equation and a vertical stretch, and purple to denote an $a < 0$ in the equation and a reflection.

---

**Lesson 6.6**

**Graphic Organizer** Provide students with an organizer that is shaped like a T-chart. On the left column of the chart students will do the work as in the left-hand column of Example 2 on page 407. On the right column students will write the steps they performed as in the right-column of Example 2. This process will help English learners clarify their understanding of the vocabulary and mechanics of solving absolute value equations.

<table>
<thead>
<tr>
<th>Work</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Group Activity** Set up groups of 2 to 4 students with English learners and English speakers, and provide them with poster paper. Assign them a problem from the Problem Solving section on pages 410 and 411 or a problem similar to those. On the poster, the students need to illustrate all their problem solving steps, including defining a variable, writing and solving an equation, and graphing the solution. Groups will present their posters to the class and the posters should be hung for the duration of the lesson for easy reference. This activity will help the English learner by creating an opportunity to present material verbally and create visual references for the classroom.

**Lesson 6.7**

**Overhead Transparency** Build on the students’ knowledge of graphing lines when introducing graphing inequalities. English learners will be more apt to grasp a new concept if it is related to prior knowledge. Create several overheads that show graphs similar to ones in Examples 2 and 3 on page 414. Have students write equations of the boundary lines by using the slope intercept method. Students should first write the equation of the boundary line and then change it to the corresponding inequality. For example, when presented with the graph for Example 2, students would write the equality $y = 4x - 3$ and then they would write the inequality $y > 4x - 3$. 

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**Lesson 7.1**

**Visual Glossary** Introduce linear systems by creating a detailed chart or poster. Post the chart to provide visual clues that the English learner can refer to while learning the concepts in the chapter. The chart should present several problems and show a connection between graphing the solution and creating a table of values. Leave space in the chart to later add the algebraic solution to each problem. For example, use a problem similar to Example 1 on page 439. Include a table of values as shown in the Exploring Algebra Activity on page 438. The table will also show the points needed to graph both equations. Color code the point of intersection in the table and on the graph to provide a visual connection between the table and the graph. Add the algebraic solution after the appropriate lesson has been introduced.

**Pairs Activity** Group students in pairs and provide them with problems similar to Exercises 12–26 on page 443. Peer communication will give English learners a chance to process concepts, procedures, and their understanding. Have one student solve half the problems, while the other student solves the other half. Then have each student check the other’s solution algebraically. Have the pairs discuss their results and go over any errors they have found. This same activity can be repeated as students learn more techniques in solving systems of equations.

**Lesson 7.2**

**Diagram** English learners will be more apt to understand methods and concepts introduced with visual clues and diagrams. While introducing the substitution method use a “circle and point” diagram as shown below to demonstrate which equation is substituted for a given variable. Including color coding will enhance the visual clue.

\[
y = \frac{3x + 2}{x + 2y} = 11
\]

\[
x + 2(3x + 2) = 11
\]

**Support for Problem Solving** To provide English learners with a tool to solve word problems like Exercises 31–38 on pages 452 and 453 supply them with double-spaced handouts of the problems. This gives them additional space, allowing them to mark the text for key phrases and information to help them in interpreting the text. Give an example of marking the text on an overhead. Also emphasize the use of diagrams and encourage student collaboration for these difficult problems.

**Lesson 7.3**

**Visual Clue** To help English learners see which variables cancel when adding or subtracting equations, color code the variables that will be eliminated. For example, in Example 1 on page 456, highlight the \(x\) terms to indicate the terms that will be eliminated. This type of visual clue allows the English learner to focus in on the main points of the lesson.

**Group Activity** By providing English learners with an opportunity to discuss ideas with peers, they will get accustomed to the language used to describe the solution process. Set up the following simple activity to provide this opportunity for these students. Create an overhead comprised of problems similar to those in Exercises 16–21 on page 459. Arrange students in groups of 2 to 4. Ask the groups to first identify which variables will be eliminated and then have each group explain how they would rearrange the equations to eliminate the designated variable.

**Lesson 7.4**

**Graphic Organizer** In solving problems in this lesson, provide students with an organizer that is shaped like a T-chart. In the left column of the chart students will show their work, and in the right column they will write the explanation of the work they performed. This process will help English learners clarify their understanding of the vocabulary and mechanics of solving systems of equations. Each step should be separated by a line.
For example, the first three steps of Example 2 on page 464 may appear as follows.

<table>
<thead>
<tr>
<th>Work</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>(4x + 5y = 35)</td>
<td>Write Equation 1 and rewrite Equation 2 so that like terms are in columns.</td>
</tr>
<tr>
<td>(-3x + 2y = -9)</td>
<td></td>
</tr>
<tr>
<td>(8x + 10y = 70)</td>
<td>Multiply Equation 1 by 2 and Equation 2 by 5 so the coefficient of (y) in each equation is 10.</td>
</tr>
<tr>
<td>(-15x + 10y = -45)</td>
<td></td>
</tr>
<tr>
<td>(8x + 10y = 70)</td>
<td>Subtract the equations.</td>
</tr>
<tr>
<td>(-(-15x + 10y = -45))</td>
<td></td>
</tr>
<tr>
<td>(23x = 115)</td>
<td></td>
</tr>
</tbody>
</table>

**Graphic Organizer** Have students create an organizer as shown below that describes the different ways to solve linear systems of equations. Include an area for an example problem and clues on when to use each problem. The concept summary on page 466 provides a basis of the information that students should have in their organizer.

<table>
<thead>
<tr>
<th>Solution Method</th>
<th>Example</th>
<th>When to Use</th>
<th>Solving Keys</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Lesson 7.5**

**Group Activity** Set up groups of 2 to 4 students with English learners and English speakers and provide them with poster paper. Give them a task card with a special type of linear system. On the poster have students solve the system algebraically and graphically. They should also write a brief analysis of their solution stating how many solutions and the types of slopes and intercepts that produced these solutions. Groups will present their posters to the class and the posters should be hung for the duration of the unit for easy reference. This activity will help the English learner by creating an opportunity to present material verbally and create visual references for the classroom.

**Pairs Activity** Pair students together, allowing English learners to clarify understanding and methods by communicating with peers. Create an overhead with several problems similar to Exercises 26–31 on page 475. Ask pairs to state how many solutions each system will have and why. To add depth to this activity, have students come to the overhead and explain their solutions.

**Lesson 7.6**

**Visual Clue** Help English learners visualize the end result of complex problems by working backwards from the solutions. Provide students with several graphs of systems of linear inequalities such as those in Guided Practice 4 and 5 on page 480. Have students write the equations that would produce the given graphs.

**Vocabulary** In Example 4 on page 480, students may have difficulty with the phrase *cannot exceed*. First show that if the length of the bat minus the weight exceeds 3, the expression can be written as \(x - y > 3\). Thus, the opposite of that, in which the length minus weight cannot exceed 3, is written as \(x - y \leq 3\).
Lesson 8.1

Visual Glossary  Help English learners gain a grasp of the vocabulary by posting a visual glossary in the classroom. Create a poster that shows a diagram of a power. Label the base and exponent as well as writing the power in expanded form. Put the poster in a place where it can be easily referred.

Also create posters diagramming the properties of powers by showing the relationship between the words and the algebra. For example, the power of a power property can be illustrated as follows.

Inquiry-Based Activity  Giving English learners an opportunity to visualize concepts will help them understand what is presented linguistically. Introduce the exponential properties by allowing the students to work through the expanded form to see the properties. Provide students with a worksheet asking them to calculate problems similar to Examples 1 and 2 on pages 505 and 506. Have them solve the problems using the expanded form. Tell them to look for patterns and write them down. Having the English learner articulate the patterns will enhance their understanding of the property.

Pairs Activity  Having English learners engage in conversations about the concepts will help them clarify their understanding. Try this exercise to work on simplifying exponential expressions. Designate one person as A and one as B. Each person will do a problem which you choose. Once they are both done with their problem, person A explains their solution while person B follows along. Then have person B explain their solution. This activity may be done with the entire class. For this activity, focus on problems such as Exercises 3–18 on page 514.

Lesson 8.2

Study Guide  Give English learners a tool to organize and synthesize material. A graphic organizer will help them structure material and concepts in a manner that is easy to access. The following type of organizer may be beneficial due to the number of definitions and properties introduced in this chapter. Have students include all the key concepts introduced.

<table>
<thead>
<tr>
<th>Key Concept</th>
<th>Example Problem</th>
<th>Answer</th>
<th>Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Product of Powers</td>
<td>$2^3 \cdot 2^2$</td>
<td>$2^5$</td>
<td>Add exponents: $3 + 2$</td>
</tr>
<tr>
<td>Quotient of Powers</td>
<td>$3^5 \div 3^2$</td>
<td>$3^3$</td>
<td>Subtract exponents: $5 - 2 = 3$</td>
</tr>
</tbody>
</table>

Practice  Understanding the order of magnitude may be difficult in the context of application problems. As an activity or quiz, prepare a list of several numbers and have students give the power of 10 closest to the numbers. After students feel comfortable with giving the order of magnitude, introduce application problems.

Lesson 8.3

Visual Clue  Provide assistance to the English learner by using as many visual clues as possible. For problems such as Exercises 50–53 on page 523, have students highlight the order of magnitude numbers involved. Have them substitute these numbers with smaller values or variables. The students should read through the problems to make sure they comprehend the question. Then have them solve the original problem.

Game  Create a simple BINGO game so students may practice the exponential properties. This will provide the English learner with a non-threatening environment in which to engage in conversation. Create a transparency with at least 36 problems similar to Exercises 3–26 on page 522. Provide each student with a $6 \times 6$ BINGO card filled with the answers to the problems on the transparency. Expose one problem at a time allowing students to solve the problem and mark the box with the answer.
Focus on Operations

Visual Glossary  Introduce the vocabulary and rules used with fractional exponents by creating a large poster for the classroom. Include examples and color coding to provide visual clues for English learners. Add terms as necessary and use the poster as an easy reference or review.

Lesson 8.4

Overhead Transparency  Introduce scientific notation with a problem that is relevant to the English learner. Make an overhead using information such as the population of a country where one student is from. Making the content relevant to the English learner will improve the chances that they become engaged with the material. By including problems that are relevant to them, you strengthen the relationship between teacher and student.

Support for Problem Solving  Set up groups of 2 to 4 students with English learners and English speakers. Provide them with poster paper and markers. Have groups solve problems similar to Exercise 53 on page 532. Groups should include all work, and work should be done using scientific notation. English learners will benefit greatly from the chance to work cooperatively in problem-solving situations. The opportunity to restate questions and explain will help them gain a deeper understanding of the material. When the posters are complete, have the groups present and post the work around the room for future reference.

Lesson 8.5

Study Guide  Giving English learners a tool to organize concepts and simplify text will enhance their comprehension. Provide students with a graph of a basic exponential growth function of the form $y = b^x$, and a few others that are of the form $y = ab^x$ for different values of $a$. Leave room on the graph so students can describe how the various graphs differ from the basic graph. This will allow students to compare and explain how graphs are influenced by different values of $a$ as in Example 3 on page 537.

Cooperative Project  Use this collaborative project based on the general compound interest formula to allow English learners the chance to interact and communicate with peers. Set up groups of 2 to 4 students with English learners and English speakers. Assign each group an amount and number of years to save for college. Have them calculate the value and share it with the class. Use the organizers below to facilitate the project.

Example of task card:

<table>
<thead>
<tr>
<th>Group</th>
<th>Initial amount</th>
<th>Time</th>
<th>Growth Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$5000</td>
<td>18 yrs</td>
<td>5 %</td>
</tr>
</tbody>
</table>

Example of student organizer:

<table>
<thead>
<tr>
<th>Initial Data</th>
<th>Interest Earned</th>
<th>Total Value</th>
<th>Graph</th>
</tr>
</thead>
</table>

Lesson 8.6

Visual Glossary  English learners will need assistance in simplifying the text and focusing on the core ideas. Create a poster showing exponential growth and decay similar to the Concept Summary on page 549. Post this in the room and refer to it throughout the unit.

Group Activity  Create several overheads that show graphs like those in Exercises 35–37 on page 552. Group the students in teams of 2 to 4 to promote interaction. Ask students to identify the type of graph, write the equation of the graph, and give other relevant information.

Focus on Functions

Visual Glossary  Have students work in groups to describe geometric sequences and arithmetic sequences in parallel. Provide them with poster paper and pens. Ask them to make a poster of two types of sequences and the tasks they have had to do with them, such as identifying the type of sequence, writing the $n$th term in a sequence, and finding the $n$th term in a sequence.
Lesson 9.1
Visual Glossary For the amount of vocabulary in this unit it will be helpful to have students construct a “word wall” at the beginning of the chapter. Give groups of 2 to 4 students one or more 8½-by-11 inch sheets of paper with a vocabulary word printed on it. Select the words from the Review Key Vocabulary at the end of the chapter. This will assist them by previewing upcoming vocabulary. Be sure to include diagrams as needed to provide a visual reference.

Visual Clue Simple notation may provide the English learner with enough of a clue to grasp an idea or concept. Color coding like terms in order to specify which terms are combined is a small clue that may have a big impact.

Lesson 9.2
Diagram Using generic rectangles as a tool to structure multiplying polynomials will be helpful for English learners. Diagrams and organizers provide visual and conceptual support for English learners. Generic rectangles like those in Example 2 on page 580 and below can be useful for multiplying binomials by monomials, binomials by binomials, or trinomials by binomials. It may also be useful later when discussing factoring of polynomials.

Lesson 9.3
Group Activity Have students investigate the patterns for the special products as a group activity. Creating a collaborative environment will give the English learner an opportunity to discuss meaning and concepts in a small group setting. Provide the students with an organizer that allows practice of multiplying polynomials as well as the opportunity to articulate mathematical patterns. The organizer can be modeled as follows.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Product</th>
<th>1st Term Pattern</th>
<th>2nd Term Pattern</th>
<th>General Pattern</th>
</tr>
</thead>
</table>

Lesson 9.4
Graphic Organizer Provide students with an organizer that is shaped like a T-chart. In the left-hand column of the chart, students should show their work as in the left side of Example 1 on page 593. In the right-hand column, students should explain the steps they performed. This process will help English learners clarify understanding of the vocabulary and mechanics of solving equations.

| Work     | Explanation |

Pairs Activity Group students in pairs and have them discuss the solutions to this activity. Peer communication will give English learners a chance to process concepts, procedures and understanding. Provide pairs with problems similar to Example 1 on page 593. Have the first student solve the problem and explain the procedure to the second student. The second will check the problem as in Example 1 and explain the procedure to the first. Have students reverse roles and repeat the process.
Lesson 9.5

Diagram Revisit the rectangles used in Lesson 9.2 to help English learners with factoring. The rectangle will provide them with a tool to organize the concept of factoring. Model for them how to simplify the problem by working on one box at a time. The top left box will be the product of the first terms. The bottom right box will be the product of the last terms. The sum of the other two boxes will be the middle term. A completed box appears as follows. Make sure to point out to students that the product of the coefficients in the two remaining boxes is the last term.

\[
\begin{array}{c|c}
 x^2 & 2x \\
 3x & 6 \\
\end{array}
\]

\[(x + 2)(x + 3)\]

Group Activity Set up groups of 2 to 4 students with English learners and English speakers and provide them with poster paper. Have groups complete Exercises 43–46 on page 605 or similar problems. Students should show all intermediary steps, include a diagram, and be prepared to present the problem to the class. This activity will help the English learner by creating an opportunity to present material verbally and create visual references for the classroom.

Lesson 9.6

Support for Problem Solving Personalize Exercise 61 on page 616 for English learners by choosing buildings or structures from their country of origin. Such personalization will help build the relationship between students and the teacher, as well as allow the English learners to feel involved with the class.

Group Activity Providing English learners with an opportunity to discuss ideas with their peers allows them to process concepts and language at a more in depth level. Setting up a simple activity can provide this opportunity for these students. Create an overhead with numerous problems similar to those in Example 2 on page 611. Begin by providing a partial solution, and then have students fill in the missing piece(s) and progress to problems where students provide the entire solution. For example, begin with a problem similar to

\[2c^2 - 7c + 3 = (2c - 1)(c - ?)\]

Lesson 9.7

Study Guide Provide English learners with materials to assist students clarify and process complex procedures and ideas. This lesson is an opportune time to review all the types of factoring. An organizer such as the one below would be a good study tool to provide for the students. Enhance the activity by allowing students to complete the organizers collaboratively and with students presenting different methods of factoring to the class. The Concept Summary on page 625 has the essential information for the study guide.

<table>
<thead>
<tr>
<th>Factoring Type</th>
<th>Clue/Indicator</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Lesson 9.8

Group Activity Use group activities to provide the English learner with the opportunity to interact and communicate about factoring polynomials completely. Create index cards with problems similar to Exercises 23–40 on page 628. Give each group a set of three cards and have them work through them. When the group is done, check their work and give them another set of three cards. Enhance this activity by having students provide solutions to the class. It is important to not provide answers and let the groups help each other find solutions.
Lesson 10.1

Study Guide  Provide students with graphs that have the parent quadratic function on it. Having an axis with the base graph will allow them to easily compare how graphs are influenced as in Example 2 on page 649. Make sure to review the meaning of a parent function by showing how other quadratic functions are derived from \( y = x^2 \).

Group Consensus Activity  Providing English learners with the opportunity to question, explain, and discuss with peers will enhance their comprehension. Create groups of 2 to 4 students and give them a set of graphs and a list of equations similar to those in Exercises 3 and 4 on page 652. Ask the groups to match the graphs and equations. Groups need to come to a consensus and be ready to discuss their results with the class.

Lesson 10.2

Visual Glossary  Create a reference poster of the properties of quadratic graphs. Include the information in the Key Concept on page 657. Post the graph in a place where English learners may easily refer to it throughout the unit. Use colors to accentuate the visual clues. For example, label \( y \)-intercepts in one color and the vertex in another. Use the corresponding colors to write the verbal descriptions.

Summary  Giving English learners an opportunity to summarize key concepts and procedures will assist them in understanding major ideas. Have each student summarize the steps for graphing a quadratic equation on a note card or in an organizer. Have the students work collaboratively to summarize their method. Ask students to share their organizers with each other at the end of the activity to ensure accuracy and allow for deeper understanding.

Focus on Functions

Group Activity  Have students work in groups of 4 to graph four quadratics functions. Give each person a graph with a factored quadratic function and the following instructions.

Step 1: Identify and plot the \( x \)-intercepts for the function. Then pass the paper clockwise.

Step 2: Use the given \( x \)-intercepts to find the axis of symmetry. Draw it on the graph. Then pass the paper clockwise.

Step 3: Use the equation and the axis of symmetry to find the vertex. After you plot it, pass the paper clockwise.

Step 4: Draw a parabola through the vertex and the \( x \)-intercepts. Then check the graph by comparing it to a graph made with a graphing calculator. Make corrections in red. Discuss corrections with your group.

Lesson 10.3

Pairs Activity  Having English learners engage in conversations about the concepts will help them clarify their understanding. Try this exercise to work on graphing quadratics. Designate one student as A and one as B. Each person will do a problem which you choose. Once they are both done with their problem, student A explains their solution while student B follows along. Then have person B explain their solution. This activity may be done with the entire class at once. For this activity, focus on problems similar to Examples 2 and 3 on page 666. Make sure that students explain their steps in drawing the graph.

Group Activity  To assist the English learners comprehend concepts, provide them with the opportunity to interact, discuss, and listen in a group situation. Form groups of 2 to 4 students with at least one English learner and one English speaker. Ask them to make a poster of a multifaceted problem similar to Exercise 51 on page 670. The groups should work together to create the poster then present it to the class. This will provide multiple opportunities for the English learner to interact in small groups helping them to process the concepts. The presentation will be another visual aid to help their understanding.

Lesson 10.4

Pairs Activity  Group students in pairs and have them discuss the solutions to this activity. Peer communication will give English learners
a chance to process concepts, procedures and understanding. Provide pairs with problems similar to Examples 2 and 3 on page 675. Have one student solve by graphing and the other student solve using algebra. When both are complete, the students should compare answers and see the connection between the solutions. Repeat this exercise with students switching their solutions methods.

**Support for Note Taking** Problems with a lot of text can pose difficulties for English learners. To help, provide them with material that helps them negotiate and simplify the text. Give them a handout that has a detailed solution of Example 5 on page 676. Allow them to use this as notes while you explain the problem on an overhead. Removing the note taking aspect will allow them to focus on the clues within the text. Require the students to highlight and make diagrams on the handout as you explain the problem. Students will be able to use this handout as a guide for similar problems.

**Lesson 10.5**

**Study Aid** Following solutions explained by written steps along the side as in Examples 2 and 3 on page 686 can be challenging for the English learner. As a study aid, provide a graphic organizer that divides the process as shown below. Students should then appropriately solve the equation on the left side.

| Write the original equation. |
| Write the equation in the form $x^2 + bx = d$. |
| Add $\left( \frac{b}{2} \right)^2$ to each side. |
| Write the left side as the square of a binomial. |
| Take square roots of each side. |
| Solve for $x$. |

**Focus on Functions**

**Graphic Organizer** Give English learners a tool to organize information in a structured format. Give them a table to use to translate reference points to their new location. For Example 1 on pages 691 you could provide the following table.

<table>
<thead>
<tr>
<th>$x_r$</th>
<th>$y_r = ax^2 = -x^2$</th>
<th>$x = x_r + h = x_r + (-2) = y_r + 3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-2$</td>
<td>$-4$</td>
<td>$-4$</td>
</tr>
<tr>
<td>$-1$</td>
<td>$-1$</td>
<td>$-3$</td>
</tr>
<tr>
<td>$0$</td>
<td>$0$</td>
<td>$-2$</td>
</tr>
<tr>
<td>$1$</td>
<td>$-1$</td>
<td>$-1$</td>
</tr>
<tr>
<td>$2$</td>
<td>$-4$</td>
<td>$0$</td>
</tr>
</tbody>
</table>

Be sure that they understand that the reference point $(x_r, y_r)$ is the point on the graph of the scaled parent graph of $y = x^2$.

**Lesson 10.6**

**Visual Glossary** Create a reference poster of the quadratic formula to be posted in the room for easy reference. Color code the $a$, $b$ and $c$ terms in the quadratic formula and in the quadratic equation.

**Group Activity** Providing English learners with the opportunity to question, explain, and discuss with peers will enhance their comprehension. Create groups of 2 to 4 students and give them an overhead transparency. Give each group the same problem and tell each group solve it by using a particular method. Make sure to choose problems that can be solved by several methods. Use the Concept Summary on page 695 to choose the methods of solution. Have the groups present their solution and note whether or not their method was efficient.

**Lesson 10.7**

**Graphic Organizer** Give English learners a tool to gather the information in a structured format that allows them to access the information more easily. Organizers will also help simplify text surrounding concepts. Use this organizer to work with discriminants.

<table>
<thead>
<tr>
<th>$a$</th>
<th>$b$</th>
<th>$c$</th>
<th>Discriminant</th>
<th>Number of Solutions</th>
<th>Graph</th>
</tr>
</thead>
</table>
Lesson 11.1
Visual Glossary Create a poster with the graph of the parent square root function. Use colors as a visual key to accentuate concepts such as intercepts, range, domain, etc. Place the poster in the classroom so students may refer to it easily. Using visual clues such as diagrams and color helps the English learner understand what is presented verbally.

Vocabulary Since the domain of radical functions is not all real numbers, take time to review the meaning of the word domain. Students may be accustomed to thinking the domain is the set of real numbers. Remind them that the domain is the set of all numbers where the function is defined.

Support for Note Taking Concepts presented with a lot of text can pose difficulties for English learners. To help students, provide them with material that assists them negotiate and simplify the text. For example, in graphing square root functions, make worksheets available with blank input/output tables and grids with axes similar to those used in Example 1 on page 734. Giving the students tools to structure the information helps them process the information more clearly and allow for a common reference point for class discussions.

Inquiry-Based Activity Provide English learners with the opportunity to question, explain, and discuss with peers to enhance their comprehension. Create groups of 2 to 4 students and have students investigate the phase shifts of square root functions. Give them an organizer similar to the one below and have them graph several functions such as Exercises 17–28 on page 738. Have students compare the graphs of the functions to the graph $y = \sqrt{x}$.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Table</th>
<th>Graph</th>
<th>Describe the shift</th>
<th>Cause of the shift?</th>
</tr>
</thead>
</table>

Lesson 11.2
Concept Organizer Allowing English learners to use organizers simplifies text for the student. This allows them to focus on the core concepts and ideas. When introducing simplifying radical expressions have students keep an organizer of the properties. Have students complete the organizer below to use as reference throughout the chapter.

<table>
<thead>
<tr>
<th>Property</th>
<th>Algebra</th>
<th>Example</th>
<th>Hints</th>
</tr>
</thead>
</table>

Game Games provide English learners with an environment to discuss ideas with peers and allows them to process concepts and language at a more in depth level. Create an overhead with numerous problems similar to Exercises 3–22 on page 747. Divide the students into teams and ask them to simplify the radicals. Award two points for the correct answer, one point for equivalent but not fully simplified answers, and zero for incorrect answers. Have teams check the work of other teams and then have them present the solutions to the class.

Graphic Organizer Facilitating the English learners ability to communicate the mathematical process will help them achieve a deeper understanding of the process. A double sided note organizer is a good tool to assist them in communicating their solution. On the organizer have them do the work on one side and explain each step on the opposite side. This tool may be beneficial for challenging problems like Exercises 55–62 on page 748. For these problems, an additional section for computing the conjugate is added.

<table>
<thead>
<tr>
<th>Work</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Write the conjugate of the denominator.</td>
<td></td>
</tr>
</tbody>
</table>
Focus on Operations

**Pairs** Have students work in pairs to simplify some of the practice problems. Let one student be student A and the other be student B. Student A gives an instruction for student B to implement. Then student B gives an instruction for student A to implement. Continue in this manner until the expression is simplified. The instructions that the students give should be similar to the descriptions given beside steps in the worked out examples on pages 751 and 752.

**Lesson 11.3**

**Pairs Activity** Pair students and provide them with problems similar to Examples 2–4 on pages 754 and 755. Have one student complete one step of a problem then pass the paper to their partner. The second student will complete the next step and return the problem to the first student. Continue this process until the problem is complete. This increased interaction helps the English learner develop understanding and can provide one-on-one tutelage if needed.

**Group Activity** Set up groups of 2 to 4 students with English learners and English speakers and provide them with poster paper. Give each group a task card with problems similar to Exercises 36–41 on pages 757 and 758. On the poster, have students solve the system algebraically and graphically. They should also write a brief analysis of their solution. Groups will present their posters to the class and the posters should be hung for the duration of the lesson for easy reference. This activity will help the English learner by creating an opportunity to present material verbally and create visual references for the classroom.

**Lesson 11.4**

**Visual Glossary** Create a poster stating the Pythagorean theorem. Use color as a visual key to accentuate concepts. Use the poster to introduce the concept and post it in the room so students may refer to it easily. Make sure to include examples of solving for an unknown length of a hypotenuse and solving for an unknown length of a leg.

**Group Activity** Group activities provide English learners with the opportunity to interact and communicate about math. Create a set of index cards with problems similar to Exercises 3–14 on page 764. Give each group three cards and have them solve the problems on the cards. When the group is done, check their work and give them another set of three cards. You may enhance this activity by awarding points for the number of sets the group works through or by having students provide solutions to the class. It is important not to provide answers and let the groups help each other find the solutions.

**Lesson 11.5**

**Support for Note Taking** Provide students with appropriate visuals to help them understand what is presented verbally. When introducing the distance and midpoint formulas, provide the students with diagrams like those in the Key Concept sections on pages 768 and 769. These diagrams will provide the English learners with tools to better understand the verbal instruction.

**Game** Create an overhead with a list of ordered pairs. Have students select at random two of the ordered pairs and then toss a coin. If heads, the students need to find distance between the two points, and if tails, the students need to find the midpoint of the line segment with the two endpoints. Continue selecting problems randomly and then have students present the solutions in front of the class.

**Support for Problem Solving** To provide English learners with a tool to negotiate the word problems of Exercises 47–52 on pages 772–774, provide students with copies of the problems typed double spaced. This allows students to have room to mark the text for key phrases and information that will help them in understanding the text. Emphasize the importance of using diagrams. English learners may be able to express the concepts better using pictures over words. Encourage student collaboration for these difficult problems.
Lesson 12.1

Visual Glossary Help English learners understand difficult vocabulary by providing visual clues throughout the room. Use the Key Vocabulary list on page 791 to create posters stating definitions and diagrams of the verbal meanings. Make sure to include past vocabulary terms used for direct variation.

Overhead Transparency Overheads and overlays can provide significant clues to meanings for the English learner. Utilize color to show the differences of the two graphs in Example 3 on page 792. These types of visual clues will help the students understand what is presented verbally. Emphasize how the negative acts as a transformation of the original graph by reflecting in the x-axis.

Lesson 12.2

Diagram Create a reference poster presenting the information of the Key Concept on page 801. Add labels showing the domain, range and asymptotes, as well as any other important information.

Pairs Activity Having English learners engage in conversations about the concepts will help them clarify their understanding. Designate one student as A and one as B. Each student will solve an assigned problem. Once they are both done solving their problems, student A explains their solution while student B follows along. Then have student B explain their solution. This activity may be done with the entire class. For this activity, focus on problems similar to Exercises 3–17 and 19–27 on page 805. In particular, have students clearly write the domain and range, and make sure students describe the transformations of the graph of \( y = \frac{1}{x} \) needed to get the given function.

Lesson 12.3

Support for Note Taking Reading solutions such as the ones for Examples 2 and 3 on page 811 can pose difficulties for English learners. Before explaining the solutions for Examples 2 and 3 on an overhead, provide students with a copy of a partial solution that they can highlight and make additional notes. By reducing the note taking aspect, students will be able to focus on the clues within the text. In presenting the steps of polynomial long division, clearly mark the general steps and procedures used. Students will then be able to use their handout as a guide for similar problems.

Group Activity Create cards with problems similar to Exercises 3–18 on page 814. Give each group a set of three cards, and have them solve the problems on the cards. When each group is done, check their work and give them another set of three cards. You may enhance this activity by awarding points for the number of sets each group successfully completes, or by having the groups present solutions to the entire class.

Focus on Operations

Pairs Have students do a couple of synthetic division problems in pairs. Let one student give the verbal description and let the other student implement it. After the step has been implemented, the student giving the instruction checks the work and makes a correction if needed. After a problem is completed, the students switch roles.

Lesson 12.4

Graphic Organizer A double-sided note organizer is a good tool to assist English learners in communicating their solution. On the organizer have them do the work on one side and explain each step on the opposite side similar to Example 2 on page 823.

<table>
<thead>
<tr>
<th>Work</th>
<th>Explanation</th>
</tr>
</thead>
</table>

Visual Clue English learners will benefit from instruction that includes visual clues to meaning. When dividing out common factors as in Example 4 on page 824, use color as a visual clue. Coloring the different factors will give the students a non-verbal clue of the mathematical concept. For example, in Example 4, \( x - 4 \) can be written in blue, \( x - 3 \) in red, and \( 4 + x \) in green.
Lesson 12.5

Pairs Activity Pair students and provide them with a problem similar to Exercises 3–10 and 13–20 on page 834. Have one student complete one step and then pass the paper to their partner. The second student will complete one step and return the problem to the first student. Continue this process until the problem is complete. As each student completes a step, they should explain what they did to their partner. The key words and phrases that should be used include “factor” and “divide out.” This increased interaction helps the English learner develop understanding and can provide one-to-one tutelage.

Support for Problem Solving Help students read the text for Exercises 33–35 on pages 835 and 836 by providing leading questions. For example, in Exercise 33, ask students to write the verbal model for the percent, and then write an equation.

Focus on Operations

Visual Clues English learners will benefit from instruction that includes visual clues to meaning. When introducing simplifying complex fractions, use color as a visual clue. Color coding the common factors will give the students a non-verbal clue of the mathematical concept.

Lesson 12.6

Group Activity Form groups of 2 to 4 students with English learners and English speakers. Provide groups with an overhead transparency. Have the groups write the solutions to problems similar to Example 5 on page 842. Groups should present their solutions and attempt to incorporate visual clues into their solutions.

Study Aid Following the multiple steps explained by written steps, as shown in Examples 4 and 5 on page 842, will be challenging for the English learner. Provide them with a study aid that simplifies the text. Give students a handout with the written steps given in a simplified form as shown. Students will need to produce the work shown on the left side but the simplified steps will provide assistance while they learn the process.

<table>
<thead>
<tr>
<th>Work</th>
<th>Step</th>
</tr>
</thead>
<tbody>
<tr>
<td>Write original expression.</td>
<td></td>
</tr>
<tr>
<td>Factor denominators.</td>
<td></td>
</tr>
<tr>
<td>Find LCD.</td>
<td></td>
</tr>
<tr>
<td>Rewrite fractions using LCD.</td>
<td></td>
</tr>
<tr>
<td>Subtract.</td>
<td></td>
</tr>
<tr>
<td>Simplify.</td>
<td></td>
</tr>
</tbody>
</table>

Lesson 12.7

Support for Problem Solving Form groups of 2 to 4 students with English learners and English speakers and provide them with poster paper. Give them a card with problems similar to Exercises 30–36 on pages 852 and 853. On the poster have students write and solve an equation. They should include a brief analysis of their solution and provide a graph when appropriate. Have groups present their posters to the class, and hang the posters as reference. This activity will help the English learner by creating an opportunity to present material verbally and create visual references for the classroom.

Graphic Organizer Give English learners a tool to organize information from this chapter in a structured format that allows them to access the information more easily. Have students complete the organizer. Include proportions, simple and complex rational expressions, as well as other important types of equations.

<table>
<thead>
<tr>
<th>Equation Type</th>
<th>Clues</th>
<th>Solution Method</th>
<th>Work</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>